Mathematical programming options for farm optimisation: A recommendation

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General Comments

Based on the results presented in "*Improved whole-farm planning for mixed-enterprise systems in Australian using a four-stage stochastic model with recourse*" (Young *et al* 2023), and the authors' experiences we provide some general insights into the optimal choice of modelling frameworks for different tasks.

The three different optimisation modelling frameworks examined are:

(i) A deterministic steady state expected weather-year framework (DSSE) (e.g. Kingwell and Pannell, 1987). In this framework the farming system is represented as a single discrete state that is statistically the expected weather-year. Representing a farm system by such a single state of nature requires use of expected inputs and outputs (e.g. the wheat yield is the average of all years). It assumes every year is the same and the finishing state equals the starting state. Thus, only strategic (or year-in year-out) management is represented and management does not change between years because there is only one branch of the decision tree being represented. This model includes 83,271 variables and 49,364 constraints.

(ii) A four-stage single-sequence stochastic programming with recourse (4-SPR) (e.g. Kingwell et al., 1991). A 4-SPR model represents the farming system as subject to a portfolio of discrete states of nature where each state represents a different type of weather-year that has separate or unique inputs and outputs to reflect different prices, weather conditions and production outcomes. All states begin from a common point that is determined by the weighted average of the end of all the weather-years, but then these states separate at various nodes during the production year to unveil the particular nature of that weather-year (Note: To minimise misrepresentation associated with the starting weighted average, the start of the weather-years is defined as the earliest season break). Once a weather-year has been identified, subsequent decisions are differentiated based on the known information about that given weather-year. For example, one node is the start of the growing season or 'break of season'. If that start is what is known colloquially as an 'early break', then after that starting point those types of weather-years can be managed differently to weather-years where the break occurs later. For example, in an early break it may be optimal to crop more area and run a higher stocking rate and vice-versa for a late break, although these decisions can only be made after the break of season is known. However, at the break of the season the subsequent conditions are uncertain (e.g. 30% chance of a poor spring and a 70% chance of a good spring). Thus, the decisions made at the break of season must factor in future uncertainty about the spring conditions. The 4-SPR model examines each possible outcome and its probability to determine the optimal decisions. These decisions are a suite of tactical adjustments made at each node that complement or adjust an overarching farm management strategy. The 4-SPR model is much greater in size, comprising 476,113 variables and 237,956 constraints.

(iii) A eight-stage multi-sequence stochastic programming with recourse framework (known as 8-SPR) (Xie and Huang, 2018). 8-SPR is similar to 4-SPR with the difference being that the discrete states represent a sequence of weather-years in equilibrium rather than a single year in equilibrium. Optimisation of management within the sequence of weather-years fully accounts for the temporal effects of management change between years. In AFO, the production data in the 8-SPR is the same as the 4-SPR for the individual weather-years. The difference is that the 8-SPR framework more accurately represents carryover management implications from the previous year. For example, if stock were sold in the previous year the current year would start with a destocked position. This version of the AFO model includes 4,571,881 variables and 2,140,700 constraints.

Firstly, the reader may be left with the impression that the DSSE framework is inferior, albeit being simpler to use. However, it should be noted that although the DSSE framework does not represent uncertainty or variation in weather-years or prices it still has the capacity to represent the biology and economics of a farming system in a very detailed way and therefore provides more accurate results than gross margins or partial budgets.

The importance of including weather variation in whole-farm bioeconomic modelling depends on factors including:

(i) The purpose of the analysis e.g. policy-making, farm planning, research prioritisation, innovation evaluation and aiding farm decision-making. For example, for a policy analysis where the focus is on ascertaining the general directional impacts on farm profit, the extra detail has less value because policy-makers are generally interested in the strategic management rather than tactical adjustments. Similarly, for assessment of some innovations, the relative difference in profit with and without the innovation is likely to be somewhat similar across the optimisation frameworks, so the detail of required farm management changes may be unnecessary to aid the decision about whether or not the innovation is worthwhile. That said, the magnitude of the profit difference associated with each optimisation framework, with and without the innovation, may be different. Where innovation users e.g. farmers or advisers, want to know more exactly the magnitude of increased profits derived from use of the innovation then a framework that describes weather-year variation may be warranted. Furthermore, if the purpose of the analysis is to provide advice to farmers on optimal management within particular weather-years regarding the innovation, then the extra detail provided by the 8-SPR or 4-SPR models may be vitally important. Janssen and van Ittersum (2007) and Reidsma et al. (2018) both similarly comment how the intended end use of a model is important for the assumptions made in a model, and the required interaction with stakeholders.

(ii) The topic of the analysis (e.g. climate change, price change, livestock productivity, pasture varieties, labour supply). For example, if the topic of the analysis is climate change, then the credibility of the analyses hinges on accurately representing changes in the probability distribution of weather-years or the types of weather-years; and how optimal farm management varies in the face of those changes. In this case, applying the 4-SPR or 8-SPR frameworks, rather than the DSSE framework would be essential.

(iii) The farm scenario being analysed e.g. financial circumstances. For example, in a finance-constrained environment it may be important to reflect the impact of last year on the opening cash balance for the current year. Hence, using the 8-SPR model is pertinent, whereas in a region that is well established with low farm debt levels, the extra detail and time required for a 8-SPR analysis may be unwarranted.

Overall, even though representing uncertainty in farm optimisation modelling requires additional user skill and time, the results provided can be substantially different from an equivalent steady state framework. Thus, in many cases the benefits of more accurate results can quickly outweigh the added cost. In the farming system outlined in this research paper, modelling without proper representation of tactical management results in foregone profit of $144 573 per year. Additionally, modern programming languages make it simple to build models with capacity to customise the level of detail represented in any individual application. Designing models that incorporate this feature allows the detail of the model to match that required for a meaningful analysis. It is also worth noting that the representation of weather-year variation becomes even more important if a farmer's attitude to risk needs to be represented; noting that a risk averse farmer's optimal management is likely to change due to weather variation.